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Borderline Cases and Bivalence

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It is generally agreed that vague predicates like 'red', 'rich', 'tall', and 'bald', have borderline cases of application. For instance, a cloth patch whose color lies midway between a definite red and a definite orange is a borderline case for 'red', and an American man five feet eleven inches in height is (arguably) a borderline case for 'tall'. The proper analysis of borderline cases is a matter of dispute, but most theorists of vagueness agree at least in the thought that borderline cases for vague predicate 'Φ' are items whose satisfaction of 'Φ' is in some sense unclear or problematic: it is unclear whether or not the patch is red, unclear whether or not the man is tall.[1] For example, Lynda Burns cites a widespread view as holding that borderline cases "are not definitely within the positive or negative extension of the predicate. … Borderline cases are seen as falling within a gap between the cases of definite application of the predicate and cases of definite application of its negation" (1995, 30). Michael Tye writes that the "concept of a borderline case is the concept of a case that is neither definitely in nor definitely out" (1994b, 18).

Reflecting this common view, the standard philosophical analysis defines borderline cases for vague predicate 'Φ' as items that are *neither definitely (clearly, determinately) Φ nor definitely not Φ*. A borderline case for 'Φ' is then also a borderline case for 'not Φ': being borderline not Φ consists in being neither definitely not Φ nor definitely not not Φ, which is equivalent to being borderline Φ. This standard analysis is usually meant to express a semantic conception of borderline cases—that is, a conception of them as arising from semantic features of a vague predicate. In that case the definiteness operator is interpreted so that the sentence '*x* is definitely Φ' is true if and only if the sentence '*x* is Φ' is true, and false if and only if '*x* is Φ' is not true, where being not true consists in being either false or neither true nor false.[2] Where *x* is a borderline case, then, the sentences '*x* is Φ' and '*x* is not Φ' are not true; they are also not false, and so are neither true nor false. Various nonclassical semantics have been introduced to capture the meanings of sentences that are neither true nor false, including such devices as supervaluations, indefinite or indeterminate values, truth-value gaps, and degrees of truth. Some of these systems invalidate Excluded Middle as well as violating bivalence, while others devise ways to preserve the classical law.[3]

Not all proponents of the standard analysis favor a semantic conception of borderline cases. For example, one much-discussed view retains a classical logic and semantics for borderline sentences by interpreting the definiteness operator epistemically: borderline cases are neither knowably Φ nor knowably not Φ. Here, borderline cases are either Φ or not Φ, and the sentences '*x* is Φ' and '*x* is not Φ' are either true or false—we just cannot know which. Even supporters of this epistemic version of the standard analysis admit that it seems counterintuitive; but agile defense, together with some trenchant criticisms of the semantic versions, have made it a contender.[4]

The details of, and arguments for, these various definitions of borderline cases are as diverse as they are many, and I will not review them here.[5] Instead I want to examine the idea, common to all versions of the standard analysis, that borderline cases are to be understood in terms of the opposition between a predicate and its *negation*. I will argue that if we give up this idea, we can formulate a robust (genuinely semantic, non-epistemic) definition of borderline cases that fits comfortably with a classical logic and semantics. On the resulting view—I call it the *Incompatibilist View*—borderline cases for vague predicate 'Φ' are not Φ, the sentence '*x* is not Φ' is true, and the sentence '*x* is Φ' is false. I assume that, all else being equal, a view that does not force abandonment of classical principles is preferable to one that does. It has seemed to most theorists of vagueness that a robust conception of borderline cases could not employ a classical semantics. I will urge that we can have it both ways.[6]

1.

As is already apparent, the Incompatibilist View will be inconsistent with all versions of the standard analysis of borderline cases. Among other things, the incompatibilist denies, whereas the standard analysis entails, that borderline cases for 'Φ' are borderline cases for 'not Φ'. Nevertheless, the Incompatibilist View may be best understood as a (revisionary) descendant of the semantic versions of the standard analysis. I want to begin, then, by setting out the basic elements of this family of views. For convenience I will call them, taken together, the 'Standard Analysis$_S$'.

First some policy about usage. To avoid scope ambiguities I am going to hyphenate the standard analysis, to give 'neither-definitely-Φ-nor-definitely-not-Φ'. For the same reason I will hyphenate the expression 'not Φ' ('not-Φ') used alone; but this practice can cause confusions of its own, so let me emphasize that 'not-Φ' will just mean whatever 'not Φ' means. Also, I will slide indifferently between formal and material modes of expression involving vague terms. For example, I will say both that '*x* is (definitely) Φ' is true and that *x* is (definitely) Φ, and both that *x* is a borderline case for the predicate 'Φ' and that *x* is a borderline case for the category or kind or property $\underline{\Phi}$ (or, more simply, that *x* is borderline Φ). There are contexts, some having to do with vagueness, in which this casual way of talking might be problematic, but it won't cause trouble here. In particular, when I speak in the material mode, I do not mean to endorse the existence of so-called metaphysical vagueness or, for that matter, any particular metaphysics of categories (kinds, properties). For present purposes, vagueness is a feature of language, or, if you prefer, of concepts.

If I understand correctly, proponents of the Standard Analysis$_S$ conceive of borderline cases in terms of a certain kind of ordering relation. They suppose that for any vague predicate 'Φ' there is some set of items that are linearly ordered with respect to Φ-ness, proceeding from an item that is definitely Φ to an item that is definitely not-Φ. Call such an ordering a *Φ-ordering*. To define 'borderline Φ', a Φ-ordering must contain at least three items: a definitely Φ item, a definitely not-Φ item, and an item that lies between the two but cannot be classified either as definitely Φ or as definitely not-Φ. (As we'll see shortly, a Φ-ordering that defines borderline cases for 'Φ' may contain more than three items—indefinitely many, in fact; and even then it may be a chain within a larger set that is only partially ordered with respect to Φ.) On the Standard Analysis$_S$, a borderline case for 'Φ' lies between the Φ item(s) and the not-Φ item(s) in a Φ-ordering, failing to be definitely classifiable with either.

Before we go further, it will be helpful to have at hand two contrasting examples of the kind of Φ-ordering that will be relevant to our discussion. First, suppose we are concerned with the richness, measured in dollars of annual income, of Americans aged forty to sixty in 2004. Then our rich-ordering contains annual incomes proceeding from one that would make such an American definitely rich, say $200,000, to one that would make him definitely middle class, hence definitely not-rich—say $50,000. Suppose further that an income of $125,000 is a borderline case, failing to be definitely classifiable either with the rich endpoint $200,000 or with the not-rich endpoint $50,000. (If you dislike these figures, feel free to substitute ones you prefer.) I am going to assume that this rich-ordering is *replete*—that is, that it contains all possible incomes that can be linearly ordered, with respect to richness, between $200,000 and $50,000. Nothing essential to my view depends upon this assumption; but for reasons that will emerge, my view must be able to accommodate a replete ordering, and given its treatment of replete orderings, its treatment of nonreplete orderings will follow easily.

Our second example will be a red-ordering of patches progressing from one that is definitely red to one that is definitely orange, hence definitely not-red. The patches midway in hue space between definite red and definite orange are borderline cases; that is, their hue fails to be definitely classifiable either with that of the red endpoint or with that of the not-red endpoint. Again I will assume that this ordering is replete: it contains all possible hues that can be linearly ordered, with respect to redness, between definite red and definite orange. For ease of discussion, let us suppose that the hues of the patches map one-to-one into the real numbers in the interval (1, 20), so that the first, definitely red patch is assigned to 1 and the last, definitely orange patch is assigned to 20. Suppose also that the borderline patches include one that is assigned to 10.[7]

As is evident in these examples, the character of the (definitely) not-Φ items in a Φ-ordering that defines borderline cases for 'Φ' is constrained in at least two ways, on the Standard Analysis$_s$. First, consider for example that the category of not-red things contains piano sonatas and prime numbers—things that could not possibly have a color (hue)—as well as orange and green items. But a borderline case for 'red' is presumably not defined by its membership in a red-ordering that proceeds from a red thing to a piano sonata, or from a red thing to a prime number. Whatever else it may be, a borderline case for 'red' is an item that has, or at least could have, a color;[8] so it is not even a *candidate* for being not-red-because-not-possibly-colored. Accordingly, advocates of the Standard Analysis$_s$ suppose that the not-Φ items, indeed all items, in a Φ-ordering that defines borderline cases for 'Φ' are also ordered on some distinct dimension D that is (at least partially) decisive of the application of 'Φ'. A red-ordering that defines borderline cases for 'red' will be an ordering on a dimension like hue, or saturation; a tall-ordering that defines borderline cases for 'tall' will be an ordering on the dimension of spatial height; a rich-ordering that defines borderline cases for 'rich' will be an ordering on (for example) the dimension of monetary amount ; and so on.[9]

The character of the (definitely) not-Φ items in a Φ-ordering that defines borderline cases for 'Φ' is constrained in a second way, on the Standard Analysis$_s$. Their character must be such as to allow for (support? justify?) a distinction between being definitely not-Φ and being borderline not-Φ. More to the point, their character as not-Φ must be such as to comport with the claim that borderline cases for 'Φ' are *not*-definitely-not-Φ. The character (hue) of the definitely not-red patches in our red-ordering #1–#20 must be such as to comport with the claim that patch #10 is not-definitely-not-red, and the character (monetary value) of the definitely not-rich incomes in our rich-ordering $200,000-$50,000 must be such as to comport with the claim that $125,000 is not-definitely-not-rich. What does this mean about their character? To pose the question in a more challenging way: why must we say that a borderline case for 'Φ' is not-definitely-not-Φ, rather than (definitely) not-Φ? We might also ask: why must the sentence '*x* is Φ' be neither true nor false in a borderline case, rather than, simply, false?

Proponents of the Standard Analysis$_s$ have offered a variety of answers to these questions. I cannot canvass all of them here, but I want to mention four that are representative and, in my view, among the most cogent.

(1) "[A] borderline case of the predicate *F* is equally a borderline case of not-*F*: it is unclear whether or not the candidate is *F*. This symmetry prevents us from simply counting a borderline *F* as not-*F*" (Keefe and Smith 1997, 7). Call this the 'argument from symmetry'.

(2) If borderline cases were not-Φ, then their status with respect to 'Φ' (and 'not-Φ') would not be indeterminate (indefinite, unclear, uncertain). There would be a "fact of the matter": borderline cases are not-Φ. Such a result runs counter to the very nature of borderline cases. Call this the 'argument from indeterminacy'.

(3) Even if we could say that borderline cases for 'Φ' are not-Φ, this would only postpone the inevitable. For then new, higher-order borderline cases would arise between the Φ items and the not-Φ (namely, borderline) items in a Φ-ordering that defines borderline cases for 'Φ'. And these new borderline cases would have to be neither-definitely-Φ-nor-definitely-not-Φ. Thus we would arrive at the Standard Analysis$_s$ at one remove, as it were. Russell puts the problem this way:

> Someone might seek to obtain precision in the use of words by saying that no word is to be applied in the penumbra, but unfortunately the penumbra itself is not accurately definable, and all the vaguenesses which apply to the primary use of words apply also when we try to fix a limit to their indubitable applicability. (1923, 87)

Call this the 'argument from higher-order borderline cases'.

(4) In judging a borderline case, we are apprised of all the relevant facts; in other words, nothing is hidden from us. Yet we can't tell that '*x* is Φ' is true and can't tell that '*x* is Φ' is false. Therefore the sentence must be neither true nor false, since if it were true we could tell that it was true and if it were false we could tell that it was false. Call this the 'argument from accessibility', the idea being that since no relevant facts are hidden, the truth or falsity of '*x* is Φ' is always accessible to competent speakers.[10]

I should note that proponents of the Standard Analysis$_s$ do typically acknowledge a sense in which a borderline Φ is not-Φ—in other words, a sense in which '*x* is not-Φ' is true in a borderline case. But the sense in question is a so-called *weak* sense of the negation, namely, a sense in which '*x* is not-Φ' is true where '*x* is Φ' is not true (that is, false or neither true nor false). On a *strong* reading of the negation, employed throughout in the Standard Analysis$_s$, '*x* is not-Φ' is true just in case '*x* is Φ' is false; and so the sentences '*x* is Φ' and '*x* is not-Φ' come out neither true nor false in a borderline case.

In what follows I will urge that by making certain adjustments to the Standard Analysis$_s$, we can plausibly define borderline cases as not-Φ; and that when we redefine them in this way, they fit comfortably with a classical semantics. We will then be in a position to respond effectively to the arguments from symmetry, indeterminacy, higher-order borderline cases, and accessibility.

2.

The view I will propose is often evident in writings by proponents of the Standard Analysis$_s$. Consider the following passages, for example:

> It may seem that *strawberry* draws boundaries, since there are no borderline cases. But this is just an accident. There could very well be, and no doubt with the advent of genetic engineering soon will be, a series of plants between strawberries and raspberries, many of them borderline for both concepts. (Sainsbury 1997, 264)

> [T]he vagueness of a vague predicate is ineradicable. Thus 'hill' is a vague predicate, in that there is no definite line between hills and mountains.

> But we could not eliminate this vagueness by introducing a new predicate, say 'eminence', to apply to those things which are neither definitely hills nor definitely mountains, since there would still remain things which were neither definitely hills nor definitely eminences, and so *ad infinitum*. (Dummett 1981, 182)

> [V]agueness is [a matter of] lacking "sharp boundaries", of dividing logical space as a blurred shadow divides the background on which it is reflected. … [This] figure equally exemplifies the idea of the borderline-case, a region falling neither in light nor shadow. (Wright 1976, 226)

> [The] concept of a borderline case is the concept of a case that is neither definitely in nor definitely out. (Tye 1994b, 18)

Notice that none of these passages characterizes borderline cases as neither-definitely-Φ-nor-definitely-not-Φ; specifically, none of them characterizes borderline cases in terms of an opposition between contradictories. Rather, they characterize borderline cases in terms of an opposition between what I'll call *incompatible* predicates, like 'raspberry' and 'strawberry', 'hills' and 'eminences', 'light' and 'shadow', 'in' and 'out'. (I will say more about incompatibility shortly.) Notice also that defenders of the Standard Analysis$_S$ often speak interchangeably in terms of contradictories and incompatibles when discussing borderline cases. For instance:

> [According to the epistemicist,] there is no genuine indeterminacy, no region of borderline cases between the red and the nonred, the bald and the nonbald, the small and the large. (Wright 1995, 133–34)

> There is a variability about the application and nonapplication of [a vague] predicate to [a borderline case]: sometimes a certain shade is described as blue while others will describe the unchanged shade, seen in the same light, as green rather than blue. Someone may be described as bald in one context and as not bald in another, though they have the same number of hairs (Burns 1995, 30).

> [A] borderline case is an object that is neither definitely *F* nor definitely not *F*. … Tarmin may have enough canine characteristics to ensure that she is definitely either a dog or a wolf, without being either definitely a dog or definitely a wolf. (McGee and McLaughlin 1994, 210)

What these passages reveal, I suggest, is that borderline cases may be definable in terms of an opposition between incompatible predicates rather than contradictory ones. My aim is to formulate such a definition.

3.

First I need to say more about the relation of incompatibility. As I will use the term, incompatible predicates 'Φ' and 'Φ*' are contrary predicates such that some linear ordering of items on a distinct dimension D, decisive of the application of both 'Φ' and 'Φ*', is both a Φ-ordering and, conversely, a Φ*-ordering. (Call such an ordering a *Φ/Φ* ordering*.) Contraries are predicates that cannot both be true, but can both be false, of the same thing. Intuitively, incompatibles are contraries that form a family: 'tall', 'average', and 'short' are incompatibles on the dimension of height; 'rich', 'middle class', and 'poor' are incompatibles on the dimension of (for instance) monetary income; 'red', 'orange', and 'green' are incompatibles on the dimension of hue; and so forth.

Now of course, even if borderline cases *can* be defined in terms of an opposition between incompatible predicates (this has not yet been shown), not all pairs of incompatibles share borderline cases. 'Rich' and 'destitute' are incompatibles but nothing is borderline between being rich and being destitute; similarly 'red' and 'green'. Just which pairs in a family of incompatibles share borderline cases seems an open question, and intuitions will diverge: does 'large' share borderline cases with 'small'? 'Red' with 'yellow'? 'Tall' with 'short'? However one resolves these questions, the point is that in order to share borderline cases, two incompatibles must be related sufficiently closely within their family. Let us say that they must be *proximate* incompatibles.

I need then to say what it is for incompatible predicates to be proximate, in a way that allows for intuitive disagreements of the sort just mentioned.[11] There may be various ways to do this, but for present purposes we can say that incompatible predicates 'Φ' and 'Φ*' are proximate just in case there is an item (or items) in a replete Φ/Φ* ordering whose value on the relevant dimension D provides some equal positive justification both for applying 'Φ' and for applying 'Φ*'.[12] In saying this I do not mean that an application of 'Φ' or of 'Φ*' would be fully justified, or a fortiori that it would be true or correct. (The import of this point will become clear.) Also, I do not mean that the value in question provides justification for applying both 'Φ' and 'Φ*', which are contraries; rather, it provides justification for both predications. So, for example, 'rich' and 'middle class' are proximate because there are incomes in a rich/middle class ordering whose values on the dimension of monetary amount provide some equal positive justification for applying 'rich' and for applying 'middle class'. 'Rich' and 'destitute' are not proximate: there can be no income in a rich/destitute ordering whose monetary value provides equal positive justification for applying 'rich' and for applying 'destitute'. Intuitively, if there is any positive justification for applying 'rich', there is no justification for applying 'destitute', and vice versa. Similarly, 'red' and 'orange' are proximate, but 'red' and 'green' are not. This way of characterizing proximate incompatibles allows for intuitive disagreements of the sort we were talking about, since competent speakers will disagree as to whether there can be items in a red/yellow ordering whose hue provides some equal positive justification for both predications, and disagree as to whether there can be items in a tall/short ordering whose spatial height provides equal justification for both predications; and so on.

With this notion of proximate incompatibility in hand, let me propose a revision to the Standard Analysis$_s$. As a first approximation, let us say that borderline cases for vague predicate 'Φ' are items that belong to a Φ/Φ* ordering but are neither definitely Φ nor definitely Φ*, where 'Φ*' is a proximate incompatible of 'Φ'. Borderline cases for 'rich' are items that belong to a rich/rich* ordering but are neither definitely rich nor definitely rich*; borderline cases for 'red' are items that belong to a red/red* ordering but are neither definitely red nor definitely red*; and so forth.[13] Or perhaps it is clearer to define first the notion of a *Φ[Φ*] borderline case*: for any proximate incompatible predicates 'Φ' and 'Φ*', *x* is a Φ[Φ*] borderline case if and only if *x* belongs to a Φ/Φ* ordering but is neither definitely Φ nor definitely Φ*. Then *x* is a borderline case for 'Φ' simpliciter if and only if there is some proximate incompatible 'Φ*' such that *x* is a Φ[Φ*] borderline case.

I said 'as a first approximation'. When borderline cases are defined by an opposition between incompatibles, the definiteness operator is otiose at least insofar as it had been introduced to avoid flat-out contradiction. So our revision of the Standard Analysis$_s$ can say simply that Φ[Φ*] borderline cases belong to a Φ/Φ* ordering but are neither Φ nor Φ*. Our proposal then is this:

(i) For any proximate incompatible predicates 'Φ' and 'Φ*', *x* is a Φ[Φ*] borderline case if and only if *x* belongs to a Φ/Φ* ordering but is neither Φ nor Φ*.

(ii) *x* is a borderline case for 'Φ' if and only if there is some proximate incompatible predicate 'Φ*' such that *x* is a Φ[Φ*] borderline case.

Call this the *Incompatibilist View* of borderline cases.

The Incompatibilist View allows us to apply a classical logic and semantics to borderline sentences. If *x* is a borderline case for 'Φ', the sentence '*x* is not-Φ' is true and the sentence '*x* is Φ' is false. For instance, the sentence '*x* is not-rich' is true, and the sentence '*x* is rich' is false, of $125,000 in our rich-ordering of incomes. In contrast to the Standard Analysis$_S$, the Incompatibilist View distinguishes, *within the category of not-rich incomes*, between those that have some claim to being classified as rich, namely the borderline cases, and those that don't, namely the middle-class incomes. Recall Keefe and Smith's argument from symmetry: "[A] borderline case of the predicate *F* is equally a borderline case of not-*F*: it is unclear whether or not the candidate is *F*. This symmetry prevents us from simply counting a borderline *F* as not-*F*" (1997, 7). On the Incompatibilist View, the symmetry is between 'Φ' and 'Φ*', not between 'Φ' and 'not-Φ'.

Some vague predicates, such as 'heap' and 'person', lack stock incompatibles in English. But even in these cases, we may suppose that there will always be some predicate that can play the required role. For example, 'smidgin' or 'tad' should work for 'heap', and 'fetus' or 'conceptus' should work for 'person'. And if there are cases where no incompatible at all exists in the language, we can coin a term for the purpose (say, 'shmeap' as an incompatible for 'heap'). It is an interesting question why some words have ready incompatibles and others don't. Part of the explanation must be just that we don't need incompatibles for lots of words, and so have not bothered to introduce them into the language. But a more interesting possibility has to do with the number of dimensions or properties that are decisive of a word's application. For example, whereas 'red' and 'rich' apply, or at least often and easily apply, on a single dimension such as hue or monetary amount, typically no single dimension is decisive of whether something is a person or a heap. Degree of gestational development, species membership, and the capacity for language use among other things may all be criterial for personhood; and as Burns (1986) observes, the spatial arrangement of a collection of sand grains, as well as their number, decides whether they constitute a heap. Thus when we construct Φ-orderings to define borderline cases for these words, we artificially isolate one among a variety of dimensions in virtue of which they apply; we isolate, say, the developmental dimension of personhood, or the linguistic dimension, or we isolate the cardinal dimension of heaphood, or the dimension of shape. Hence we should not be surprised if incompatible terms are not readily available on each of these dimensions.

Three points need emphasis. First, there is nothing epistemic about the Incompatibilist View. As on the Standard Analysis$_s$, borderline cases arise from semantic features of the predicates involved: borderline cases arise because the relevant 'Φ' and 'Φ*' are only contraries, and their extensions are not together exhaustive over the range of values in a Φ/Φ* ordering. One could say that Φ[Φ*] borderline cases "fall within the gap" between the extensions of 'Φ' and 'Φ*'. Second, the incompatibilist's 'not-Φ' is not the weak negation I mentioned earlier: '*x* is not-Φ' is true just in case '*x* is Φ' is false. The incompatibilist's 'not-Φ' is a classical negation, and a classical semantics does not distinguish between strong and weak negations. Further, any distinction between so-called internal and external negations collapses: '*x* is not-Φ' and 'It is not the case that *x* is Φ' mean the same.

Third, I have not claimed that on the Incompatibilist View, the predicates 'not-rich' and 'not-red' lack borderline cases; I have claimed only that borderline cases for 'rich' and for 'red' are not borderline cases for 'not-rich' and for 'not-red', contrary to the Standard Analysis$_s$ (indeed, contrary to all versions of the standard analysis).[14] That said, what *does* the incompatibilist say about 'not-rich' and 'not-red'? Consider for example that the converse of our rich-ordering $200,000–$50,000 is a not-rich-ordering extending from a not-rich income to a not-not-rich one. Does 'not-rich' have borderline cases in this ordering? What about 'not-not-rich'? Proponents of the Standard Analysis$_s$ would presumably say that 'not-rich' and 'not-not-rich' share borderline cases that are neither-definitely-not-rich-nor-definitely-not-not-rich. The incompatibilist, on the other hand, will reply that these predicates do not share borderline cases, since they are contradictories, but 'not-not-rich' shares borderline cases with its proximate incompatible 'middle class'. The latter borderline cases belong to a not-not-rich/middle class ordering but are neither not-not-rich nor middle class. In general, borderline cases for 'not-not-rich' are neither not-not-rich nor not-not-rich*—which is, of course, the same as being neither rich nor rich*. Unsurprisingly, the borderline cases for 'not-not-rich' are just the borderline cases for 'rich'. The sentence '*x* is not-not-rich' ('*x* is Φ') is false of these borderline cases, as is the sentence '*x* is rich'; and the sentence '*x* is not-not-not-rich' ('*x* is not-Φ') is true, as is the sentence '*x* is not-rich'.

If 'not-rich' does not share borderline cases with 'not-not-rich', does this mean that not-rich has no borderline cases at all? Surely, someone might say, the incomes in the ordering $200,000–$50,000 could be sorted into categories other than **rich** (or **not-not-rich**), **middle class**, and **rich[middle class] borderline**. For instance, couldn't the category **not-rich** have as a proximate incompatible the category **super-rich**, and the intervening (rich but not-super-rich) incomes be their shared borderline cases? These borderline cases for 'not-rich' would be incomes that belong to a not-rich/super-rich ordering but are neither not-rich nor super-rich. That is, they would be not-not-rich and not-super-rich—that is, rich but not-super-rich.

The first point to make in reply here is that 'not-rich' and 'super-rich' probably are not proximate incompatibles, since it is hard to see how there could be an income in a not-rich/super-rich ordering whose monetary value provides some equal positive justification for applying 'not-rich' and for applying 'super-rich'. And if 'not-rich' and 'super-rich' are not proximate incompatibles, they do not share borderline cases. But never mind about that. Suppose the incomes could be sorted so that 'not-rich' has a proximate incompatible in the ordering in question. Then as far as the incompatibilist is concerned, 'not-rich' has borderline cases in that ordering. Strictly speaking, all the incompatibilist claims is that a predicate has (or can have) borderline cases if and only if it has a proximate incompatible. She does not attempt to answer the further question of whether, or why, a predicate has a proximate incompatible in any given case.[15]

In the next section we will take up some potential objections to the Incompatibilist View. Before turning to those, I want to emphasize that while I have chosen to develop the Incompatibilist View as a revision of the Standard Analysis$_s$, this way of developing it is not essential. I think it is the most illuminating way, but only the most illuminating. Even if the Incompatibilist View is not properly conceived as a descendant of the Standard Analysis$_s$, it will stand as an alternative semantic definition of borderline cases that can be considered on independent grounds.

4.

Now for some possible objections. I will frame these as arising from the viewpoint of the Standard Analysis$_s$, but similar worries might be raised by advocates of other versions of the standard analysis as well.

1. Perhaps defenders of the Standard Analysis$_s$ (our present opponents) will contend that their strong 'not-Φ' is just equivalent to the incompatible predicate 'Φ*' in a Φ/Φ* ordering; and so the Incompatibilist View collapses into the Standard Analysis$_s$.[16] For instance, they may contend that within the span of a red/orange ordering, all not-red items are orange; hence since all orange items are not-red, 'not-red' and 'orange' are equivalent. This cannot be right, however, for on the Incompatibilist View the predicates 'red' and 'orange' are contraries—within a red[orange] ordering and without. Alternatively, our opponents might argue that 'not-Φ' in the Standard Analysis$_s$ is equivalent to the disjunction of all (possible?) incompatibles of 'Φ'. For example, maybe 'not-red' is equivalent to the disjunction 'orange or red-orange or orange-yellow or yellow or … or green or … and so forth'. (Let us use 'red-orange' to name the incompatible hue category between *red* and *orange*, 'orange-yellow' the hue category between *orange* and *yellow*, and so on.)[17] This cannot be right either, though, for then no item of any color would be a borderline case for 'red': any colored object satisfies some hue predicate or other, hence satisfies either 'red' or one of its incompatibles. The borderline cases in our red/orange ordering, for example, are red-orange. The idea that 'not-red' is equivalent to the disjunction of all *proximate* incompatibles of 'red' fares no better, since every borderline case for 'red' satisfies some or other proximate incompatible of 'red'—'red-orange', or 'red-violet', and so forth. (For that matter, on this proposal green things would count as borderline cases for 'red'.) The problem isn't fixed by saying that, *qua borderline case*, a borderline case for 'red' does fail to satisfy any hue predicate at all; for by that reasoning every borderline case of *any* hue, *qua borderline case*, would be a borderline case for 'red'. It also doesn't help to suppose that 'not-Φ' in the Standard Analysis$_s$ is equivalent to the disjunction of all (possible?) incompatibles of 'red' in a red/orange ordering; for any item in a red/orange ordering satisfies either 'red' or one of its incompatibles in that ordering. Similarly for the disjunction of all proximate incompatibles of 'red' in a red/orange ordering. As far as I can see, other analogous attempts to push the Incompatibilist View into the Standard Analysis$_s$ will face the same sorts of difficulties.

2. Remembering Russell's admonition about higher-order borderline cases, friends of the Standard Analysis$_s$ might suppose that the incompatibilist has merely postponed the inevitable. If there can be items that are borderline between 'Φ' and 'Φ*' in a Φ/Φ* ordering, then surely there can be items that are borderline between 'Φ' and 'Φ[Φ*] borderline'. And since the former items are not-Φ, won't the latter items be borderline between 'Φ' and 'not-Φ'? For example, if there can be incomes that are borderline between 'rich' and 'middle class' in our ordering \$200,000–\$50,000, then there can be incomes that are borderline between 'rich' and 'rich[middle class] borderline', that is, between 'rich' and 'rich[middle class] not-rich-and-not-middle-class'. Won't the incompatibilist have to say that these second-order borderline cases are neither-definitely-rich-nor-definitely-not-rich? Won't the Incompatibilist View now collapse into the Standard Analysis$_s$ at one remove, as it were?

I will answer this question (negatively) in a moment. First let me supply some background. Many advocates of the Standard Analysis$_s$ suppose that a Φ-ordering that defines borderline cases for 'Φ' also contains indefinitely many higher-order borderline cases. If first-order borderline cases are neither-definitely-Φ-nor-definitely-not-Φ, second-order borderline cases are neither-definitely-definitely-Φ-nor-definitely-not-definitely-Φ, third-order borderline cases are neither-definitely-definitely-definitely-Φ-nor-definitely-not-definitely-definitely-Φ, and so on *ad indefinitum.* (To keep things simple, I mention only one class of borderline cases of each order.) Such a hierarchy is supposed to be essential to the vagueness of 'Φ', ensuring the absence of any sharp boundaries in the predicate's application.[18] In effect, then, these defenders of the Standard Analysis$_s$ will present the incompatibilist with a dilemma: either higher-order borderline cases can be defined between 'Φ' and 'not-Φ', or else 'Φ' has sharp boundaries of application.

The incompatibilist will reject the dilemma as false. As regards the first horn, there are no higher-order borderline cases: borderline cases, higher-order or otherwise, are not defined between contradictories. The indefinitely many higher-order borderline cases posited on the Standard Analysis$_s$ are just the indefinitely many Φ[Φ*] borderline cases in a replete Φ/Φ* ordering. (If a replete ordering turns out not to contain indefinitely many members, then it does not contain indefinitely many Φ[Φ*] borderline cases—but of course it does not contain indefinitely many higher-order borderline cases, either.) In our example, putative higher-order borderline cases for 'rich' are just the indefinitely many rich[middle class] borderline incomes in a replete rich/middle class ordering. Generally speaking, on the Incompatibilist View, any Φ-ordering that defines borderline cases for 'Φ' is exhausted by three categories of items: Φ items, Φ* items for relevant 'Φ*', and Φ[Φ*] borderline cases.[19]

From the vantage of the Standard Analysis$_s$, the existence of higher-order borderline cases may seem inevitable. Keefe and Smith write:

> It is widely recognized … that the borderline cases of a vague predicate are not sharply bounded. This is often taken to imply that there is a hierarchy of borderline cases. … The lack of sharp boundaries between the *F*s and the not-*F*s shows that there are values of *x* for which it is indeterminate whether *x* is *F* and equally indeterminate whether it is not-*F*. So similarly, the lack of a sharp boundary between the clear *F*s and the borderline *F*s must imply that there are borderline borderline cases—values of *x* for which it is indeterminate whether *F*x is borderline [and so on *ad indefinitum*]. … Any putative theory of vagueness must accommodate the apparent lack of sharp boundaries to the borderline cases and address the issue of higher-order vagueness. (1999, 15–16)

The moral of the incompatibilist story, however, is that higher-order borderline cases seem inevitable only if borderline cases are defined in terms of contradictories. On the Incompatibilist View, borderline cases are not defined between the clear *F*s and the borderline *F*s because the borderline *F*s are not-*F*.

One might have thought that 'rich[middle class]borderline' is a proximate incompatible of 'rich', and so shares (second-order) borderline cases with it. But recall the definition of 'incompatible': incompatible predicates 'Φ' and 'Φ*' are contraries such that some linear ordering of values on a distinct dimension D, decisive of the application of both 'Φ' and 'Φ*', is both a Φ-ordering and conversely a Φ*-ordering. Simply put, 'Φ' and 'Φ*' must be contraries *in a Φ/Φ* ordering*. In order to be incompatibles, then, 'rich' and 'rich[middle class] borderline' would need to be contraries in that segment of the rich/middle class ordering that extends from 'rich' at $200,000 to 'rich[middle class] borderline' at, say, $125,000. And they are not: whereas they are contraries in the ordering from 'rich' to 'middle class', they are *contradictories* in the segment from 'rich' to 'rich[middle class] borderline'.[20] Intuitively, 'rich' and 'rich[middle class] borderline' are contraries in the ordering from 'rich' to 'middle class' only by dint of the presence of the middle-class incomes; whereas in the segment from 'rich' to 'rich[middle class] borderline', the middle-class incomes have dropped out.

Alternatively, one might imagine that second-order borderline cases could be defined in an ordering from 'rich' to 'rich[middle class] borderline' that extends from $200,000 to $125,000 but is distinct from, in other words is not simply a segment of, the rich/middle class ordering $200,000–$50,000. The idea would be that in this distinct shorter (still replete) ordering, the extension of 'rich' shrinks to make room for new borderline cases; and so the predicate 'rich[middle class] borderline' now *is* an incompatible of 'rich'. (Since there would be no middle class-incomes in this shorter ordering, we can characterize the rich[middle class] borderline cases therein as, simply, *rich[middle class] not-rich.*) In other words, some incomes that are rich in the rich/middle class ordering would be second-order borderline cases for 'rich' in the new shorter ordering: they would belong to a rich/rich[middle class] not-rich ordering but be *neither rich nor rich[middle class] not-rich.* Third-order borderline cases would be defined in a still shorter rich-ordering from 'rich' at $200,000 to 'second-order borderline' or 'rich[rich[middle class] not-rich] not-rich' at, say, $160,000. And so on *ad indefinitum.* Unsurprisingly, the extensions of 'rich' and 'not-rich' would differ, shifting together, in the different rich-orderings within this hierarchy. Thus the indefinite hierarchy of higher-order borderline cases posited by the Standard Analysis$_S$ would be replaced by an indefinite hierarchy of higher-order Φ-orderings associated with any given Φ/Φ^* ordering, each defining a class of higher-order borderline cases. (Figure 1 below, if ungrammatical, may help to convey the idea. 'B1', 'B2', and 'B3' stand for first-, second-, and third-order borderline cases respectively.)

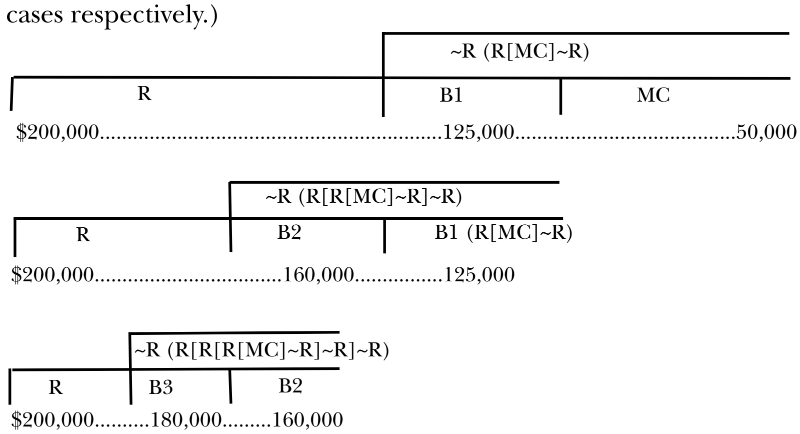

Figure 1.

I think such a view is untenable for a variety of reasons. We needn't worry about those here, though, since even if higher-order borderline cases could be generated in the way just described, they would pose no threat to the Incompatibilist View. For Russell's worry to get hold, the borderline cases of any given order would need to lie between the rich incomes and the not-rich incomes *in that ordering*—in other words, *in the ordering in which they are defined as borderline cases*. We might express the point generally by saying that in order for Russell's worry to get hold, the n^{th}-order borderline cases for 'Φ' would need to lie between the items that are Φ and the items that are not-Φ *in the n^{th}-order ordering*; more simply, the n^{th}-order borderline cases would need to lie between the nth-order Φ items and the nth-order not-Φ items. For example, the second-order borderline rich incomes would need to lie between the second-order rich incomes and the second-order not-rich incomes. But they don't: as is clear in figure 1, the second-order borderline cases lie between the second-order rich incomes and the *first-order* not-rich incomes. In the second-order ordering, the second-order borderline cases are, like all borderline cases for 'rich', not-rich. They are second-order not-rich, or rich[rich[middle class] not-rich] not-rich. Analogously at higher orders. In any Φ-ordering that defines borderline cases for 'Φ', the borderline cases are not-Φ.

All of this said, the Incompatibilist View does provide for something very like higher-order borderline cases. In addition to our rich/middle class ordering \$200,000–\$50,000, there is a distinct, rich/upper middle class ordering extending from \$200,000 to \$125,000, containing rich[upper middle class] borderline cases; and also a rich/upper upper middle class ordering extending from \$200,000 to, say, \$160,000, containing rich[upper upper middle class] borderline cases; and so on, presumably, *ad indefinitum.*[21] This progression of increasingly shorter rich-orderings, employing new and increasingly finer-grained incompatible predicates, may capture much of what advocates of higher-order borderline cases have had in mind—absent anything higher order. (See figure 2 below. In contrast to the situation depicted in figure 1, here the different orderings cross-classify one another. The introduction of a new incompatible predicate in each ordering—'upper middle class', 'upper upper middle class', and so on—forces a new classification.)[22]

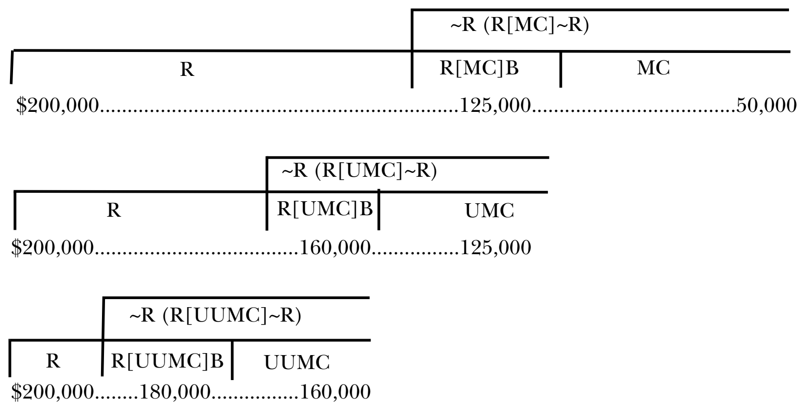

Figure 2.

So much for the first horn of the dilemma. What about the second? If the Incompatibilist View does not allow for higher-order borderline cases, doesn't it follow that 'Φ' has sharp boundaries of application? Specifically, won't there be a sharp boundary between the Φ items and the Φ[Φ*] borderline cases, and another between the Φ[Φ*] borderline cases and the Φ* items?

The answer is 'no', though the reasons why, having to do with the semantics of vague predicates generally, lie beyond the scope of our discussion here.[23] For present purposes it will suffice to observe that defenders of the Standard Analysis$_s$ with higher-order borderline cases are not entitled to raise this objection against the incompatibilist, for their view equally yields a tripartite classification: any Φ-ordering that defines borderline cases for 'Φ' is exhausted by three categories of items: Φ items, not-Φ items, and borderline cases of one order or another.[24] Moreover, on both accounts the supposed sharp boundaries would flank the region of borderline cases: if there are no sharp boundaries among the various orders of borderline cases on the Standard Analysis$_s$, there are equally no sharp boundaries within the class of Φ[Φ*] borderline cases on the Incompatibilist View. Thus the latter view seems no more likely to install sharp boundaries than the former.

3. The Incompatibilist View does not acknowledge even "first-order" borderline cases between 'Φ' and 'not-Φ' in a Φ/Φ* ordering. Doesn't it follow from this alone that 'Φ' has sharp boundaries of application?

Again I think the answer is 'no', and again some of the principal reasons lie beyond the scope of this paper. For now it will be enough to point out that the claim that an absence of borderline cases between 'Φ' and 'not-Φ' results in a sharp boundary between the two predicates presupposes that borderline cases for 'Φ' are defined in terms of 'Φ' and 'not-Φ'—defined but (necessarily) uninstantiated. According to the incompatibilist, however, borderline cases are not defined in terms of contradictories. From the incompatibilist's point of view, claiming that the predicate 'rich' has sharp boundaries because there are no borderline cases between 'rich' and 'not-rich' is rather like claiming that 'rich' has sharp boundaries because there are no borderline cases between 'rich' and 'piano sonata'. 'Rich' and 'piano sonata' are not the sort of terms that can share borderline cases; the same goes, if for different reasons, for 'rich' and 'not-rich'. Also, again, even if a sharp boundary were to obtain between the Φ items and the not-Φ (borderline) items on the Incompatibilist View, an equally sharp boundary would obtain between the Φ items and the variously ordered borderline cases on the Standard Analysis$_S$.

4. Doesn't the incompatibilist fail to capture the indeterminacy (indefiniteness, uncertainty) associated with borderline cases? On the Incompatibilist View, borderline cases for 'Φ' are not-Φ and not-Φ*. Where is the indeterminacy in that?

Of course, if 'indeterminacy' just means something expressible only in terms of an opposition between contradictories, or only in terms of a semantically (or epistemically) interpreted definiteness operator, then the Incompatibilist View does not capture it. The incompatibilist doesn't think that borderline cases have such a feature. Nevertheless, she does agree that borderline cases occupy an indeterminate status, where 'indeterminate' is understood in a different way. Actually, the incompatibilist will distinguish between the indeterminacy or indefiniteness of borderline cases, on the one hand, and the uncertainty associated with borderline cases, on the other. Let me explain.

On the Incompatibilist View, the claim that a borderline case belongs to a Φ/Φ* ordering but is neither Φ nor Φ* just is a way of saying that the category membership of a borderline case is indeterminate. For example, to say that patch #10 belongs to a red/orange ordering but is neither red nor orange is to say that its hue is no more determinate than that: its hue lies in the range from red to orange, but by the nature of the case no further specification can be made, no particular hue can be ascribed—given the available response categories (predicates). In contrast, if the patch were red or were orange, then its hue would be determinate. One might say that where **red** and **orange** are the only categories available, patch #10 has no particular, that is, no determinate, hue. (It "falls within the gap" between red and orange.) This is not to say that #10 has no determinate hue *at all*; #10 is, for example, red-orange. But insofar as #10 is a borderline case for 'red' and 'orange', the category **red-orange** is not available. Similarly, given that **rich** and **middle class** are the only categories available, $125,000 belongs to no particular, no determinate, income category.

The uncertainty associated with borderline cases, on the other hand, may simply be a feeling that is provoked by them; 'discomfort' would be a better name. We naturally feel uncomfortable when confronted with a patch like #10, which has no determinate hue, or an income like $125,000, which belongs to no determinate income category. We feel uncomfortable because, given the available alternatives—'red' and 'orange', 'rich' and 'middle class'—we lack the linguistic resources to classify the items in question. Patch #10 and $125,000 are classificatory loose ends ; and classificatory loose ends make us uncomfortable. (Our discomfort must be most acute when the predicates at issue are of social or other importance, like 'person' and 'fetus', or 'adult' and 'juvenile'.) In addition, the restriction to two predicates that makes borderline cases possible may carry with it the pragmatic implication that one of the two terms ought to apply: where (for example) 'rich' and 'middle class' are the only alternatives, there is a certain pressure, as it were, to apply one or the other. (And of course a borderline case for 'rich' and 'middle class' does belong to a rich/middle class ordering, so it is at least a *candidate* both for being rich and for being middle class. It is not, for example, a destitute income.) This implication, this pressure, may contribute to our discomfort in a borderline case where neither predicate applies.[25]

5. If no definiteness operator occurs in the analysis of borderline cases, and the indeterminacy of, and uncertainty associated with, borderline cases are to be understood in the manner described above, what do ordinary speakers mean when they use the term 'definitely' (if indeed they do) in connection with borderline cases? What do ordinary speakers mean by saying that an item is, or is not, definitely Φ?

There is more to be said on this topic than I can say here, not least because some work will be required to discover just how, if at all, the word 'definitely' is used in connection with borderline cases in ordinary speech. For now let me simply broach the idea that the ordinary 'definitely' is a pragmatic operator used to express (the presence or absence of) the feeling of discomfort caused by borderline cases. When we say that an item is definitely Φ, we are expressing our feeling of being comfortable about applying the predicate; and when we say that an item is not-definitely-Φ, we are expressing our feeling of being uncomfortable about applying the predicate. So we might well say of a red[orange] borderline case that it is not definitely red and not definitely orange, expressing feelings of being uncomfortable about applying 'red' and uncomfortable about applying 'orange'; or say of a rich[middle class] borderline case that it is not definitely rich and not definitely middle class. Presumably we say that something is (or is not) definitely Φ only in contexts where the possibility of borderline cases, and the attendant discomfort, is already alive; apart from that possibility, use of 'definitely' is pragmatically inappropriate.[26] Again, part of what makes us uncomfortable about a borderline case is a restricted (impoverished?) set of response categories. And of course, if we are linguistically competent, we don't feel uncomfortable unless the item in question belongs to the relevant type of Φ/Φ* ordering.

As I say, the role of 'definitely' in ordinary speech deserves a good deal more attention; but I will leave it at that for present purposes.

5.

Let us now review where we stand with respect to the arguments from symmetry, indeterminacy, higher-order borderline cases, and accessibility. These arguments, recall, have been offered by defenders of the Standard Analysis$_s$ against the idea that borderline cases for 'Φ' are not-Φ. How does the incompatibilist respond?

(1) In response to the argument from symmetry, the incompatibilist grants that there is a symmetry but locates it between 'Φ' and a proximate incompatible 'Φ*', rather than between 'Φ' and 'not-Φ'. (2) Regarding the indeterminacy of borderline cases, the incompatibilist grants that borderline cases occupy an indeterminate status but defines the indeterminacy in the different way outlined in section 4. The main idea is that a borderline case "falls within the gap" between available incompatible categories and hence belongs to no particular, that is, no determinate, category of the relevant sort. (3) The worry about higher-order borderline cases is allayed by the fact that such cases cannot occur on the Incompatibilist View.[27] This is because borderline cases are defined between incompatibles, not contradictories.

(4) To handle the accessibility argument, the incompatibilist grants that if 'x is Φ' is true we can tell that it is true, and if 'x is Φ' is false we can tell that it is false; but she contends that 'x is Φ' is false in a borderline case.[28]

So far in this paper, I have not argued that the Incompatibilist View provides the correct definition of borderline cases, or that any other definition is wrong. My goal has been only to formulate the Incompatibilist View and fend off some initial objections. Before closing, however, I want to mention briefly some potential advantages of my view over the Standard Analysis$_s$, keeping in mind that what I am about to say does not yet constitute a definitive case.

(1) As I have claimed, the Incompatibilist View appears to provide a genuinely semantic analysis of borderline cases without having to give up bivalence. (Presumably this is also an advantage over the epistemic version of the standard analysis, which preserves a classical semantics at the cost of gutting the phenomena of vagueness and borderline cases. Of course, in preserving bivalence, the Incompatibilist View undercuts much of the motivation for epistemicism anyway.)

(2) The Incompatibilist View dispenses with the definiteness operator and so is freed from the complications that come with that device.[29] Any potential drawbacks of eliminating the operator are mitigated by the fact that the word 'definitely', as used (multifariously) by advocates of the Standard Analysis$_s$, is a term of art with no clear basis in commonsense. (I myself have not heard an ordinary speaker refer to a borderline case as neither-definitely-Φ-nor-definitely-not-Φ.) Indeed Keefe, herself a patron of the definiteness operator, warns against

> constructing an account of *D* via one's theory and assuming that it corresponds exactly to a pre-theoretic notion. … The ordinary use and apprehension of 'definitely' may well not straightforwardly conform to the kind of formal theory of the *D* operator that theorists seek. Intuitions about the operator may be inconsistent … [and] anyway, the consequences of the theory of *D* will outstrip the consequences we would expect given only our intuitions about 'definitely'. … It is thus reasonable, and perhaps necessary, to give 'definitely' a technical sense that depends on and is dictated by the theory of vagueness offered for the *D*-free part of language. (2000, 30)

In general, the Standard Analysis$_s$ does not appear to enjoy any intuitive advantage over the Incompatibilist View. Defenders of the Standard Analysis$_s$ like to point out that ordinary English often expresses an absence of borderline cases by asserting Excluded Middle: 'Either he *is* your father or he *isn't*', 'Either you *are* pregnant or you're *not*'. But it uses incompatibles too: 'You're either with us or against us', 'Take it or leave it', 'It's a black and white situation', 'You'll either love it or hate it', 'He'll either sink or swim'. (See how the latter statements are trivialized if the incompatibles are replaced by contradictories: 'It's a black and nonblack situation', 'You'll either love it or not love it', 'He'll either sink or not sink', and so forth.) An anonymous referee for the *Philosophical Review* objects to the Incompatibilist View: "It's hard to see how someone could believe that *a* is a borderline case of *F* without feeling some kind of uncertainty when posed with the question whether *a* is *F* or not."[30] This may be true, but I would suggest that the uncertainty owes instead to an implicit question *whether a is F or F**, for some (perhaps not readily available) proximate incompatible 'F*'. 'Is the carpet red or orange?' 'Is Tarmin a dog or a wolf?' 'Are you with us or against us? In a case where no proximate incompatible predicate is readily available, then since ordinary language has no variable term like 'F*', we may resort to the negation 'not-F'. It may be that if we withhold the predicate 'not-Φ' from a borderline case, what we actually mean to withhold is some proximate incompatible of 'Φ'. (The sometimes lack of a ready incompatible may help to explain why proponents of the Standard Analysis$_s$ have thought that the predicate 'not-Φ' must be withheld from a borderline case, and why they tend to slide between using contradictories and using incompatibles when characterizing borderline cases.)

(3) Since it rules out the possibility of higher-order borderline cases, the Incompatibilist View is freed of the complications that come with that notion as well. As with the definiteness operator, any potential drawbacks of eliminating higher-order borderline cases are mitigated by the fact that the notion has no obvious basis in commonsense. I have not heard an ordinary speaker call something a borderline case of a borderline case, much less a borderline borderline borderline etc. case; in fact I doubt that ordinary speakers would make much sense of the idea. (Maybe higher-order borderline cases are an artifact of the standard analysis.)[31]

(4) By defining borderline cases in terms of incompatibles rather than contradictories, the Incompatibilist View avoids the following, counterintuitive consequence of the Standard Analysis$_s$. Consider again our red/orange ordering of patches #1–#20. As we saw earlier, the strong 'not-Φ' of the Standard Analysis$_s$ is not equivalent to any incompatible or any disjunction of incompatibles of 'Φ'—in a given Φ/Φ* ordering or otherwise. In particular, 'not-red' is not equivalent to any

incompatible 'red*' or any disjunction of such incompatibles. It would seem to follow then, on the Standard Analysis$_S$, that the extension of 'not-red' in the red/orange ordering must be larger, must contain more hues, than the extension of 'orange'. (It certainly cannot contain fewer.) By the same token, since 'not-orange' and 'red' cannot just be equivalent, the extension of 'not-orange' must be larger than the extension of 'red'. But then the class of patches that are neither-definitely-red-nor-definitely-not-red is not coextensive with the class of patches that are neither-definitely-orange-nor-definitely-not-orange. Such a result seems implausible at best. (Figure 3 below illustrates one possible way in which the values in question might be distributed. The smaller sections flanking the borderline cases include any higher-order borderline cases.)

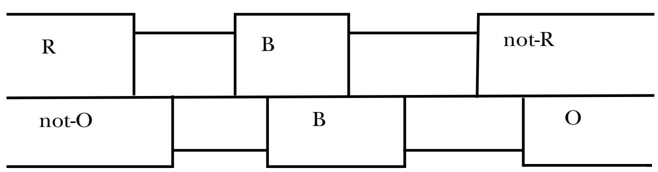

Figure 3.

More perplexing still, insofar as proponents of the Standard Analysis$_S$ do sometimes characterize borderline cases in terms of incompatible predicates, there would seem to be a third distinct class of borderline cases in a Φ/Φ* ordering. Where exactly are these items supposed to be located? For example, where exactly are the red[orange] borderline cases in our red/orange ordering, on the Standard Analysis$_S$? Our opponents owe us an answer.

In contrast, the Incompatibilist View does not have these undesirable consequences. In keeping with commonsense intuition, only one class of borderline cases is defined in a Φ-ordering that defines borderline cases for 'Φ', and the items (values) contained therein are borderline for both of the opposed predicates. All borderline cases defined in a given Φ/Φ* ordering are Φ[Φ*] borderline and, equivalently, Φ*[Φ] borderline. The borderline cases for 'red' in a red/orange ordering just are the borderline cases for 'orange' in that ordering; red[orange] borderline cases just are orange[red] borderline cases, and rich[middle class] borderline cases just are middle class[rich] borderline cases.

In retrospect, the idea of a symmetry between contradictories in the definition of a borderline case should have seemed unlikely, since the extensions of a (vague) predicate and its contradictory are not in general symmetrical. At least as far as atomic predicates like ‘red’, ‘rich’, ‘tall’, and ‘bald’ are concerned, the extension of ‘not-Φ’ is significantly larger (and more heterogeneous) than the extension of ‘Φ’. There are many more not-red things in the universe than red ones, many more not-rich things than rich ones. Is there any nontendentious reason to think that the situation is different within a Φ-ordering that defines borderline cases for ‘Φ’? Granted, such an ordering provides a restricted domain, but some compelling argument would be needed nonetheless.

Again, the preceding considerations do not constitute a definitive case for the Incompatibilist View; but I hope I have said enough to warrant optimism that such a case can be made.

6.

I have developed the Incompatibilist View as a revision of the Standard Analysis$_S$ of borderline cases, but as I indicated earlier, this way of developing it is not essential. Quite apart from any reference to, or criticisms of, the Standard Analysis$_S$, the Incompatibilist View can be considered independently as a robust analysis of borderline cases that preserves bivalence . Most theorists of vagueness have thought that in order to apply a classical logic and semantics to borderline cases, one had to forego a genuinely semantic conception of them. I have argued that the sacrifice is not necessary.

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Notes

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[1] For reasons that will emerge, I use 'Φ' in the manner of an object-linguistic predicate: 'Φ' is satisfied by items that are Φ. It is likely that many parts of natural language speech are vague in the "soritical" sense under discussion here, including verbs, adverbs, and ordinary proper names; but predicates are the cases of greatest philosophical interest and I will focus on those. My expectation is that the account I propose for vague predicates could be extended to other vague terms.

[2] Here I simplify and disregard, for the moment, the possibility of various forms of higher-order borderline cases; I say more in section 4.

[3] See for example Machina 1976 and Fine 1975, respectively. Contrary to a common practice, I do not suppose that a semantic view of borderline cases just is a version of the standard analysis that interprets the definiteness operator semantically (as in the text above). I emphasize this point because the view I will propose is genuinely semantic—it conceives borderline cases as arising from semantic features of a vague predicate—but it does not employ a definiteness operator.

[4] See for example Sorensen 1988, 1994, Williamson 1994. Burns (1991) follows David Lewis (for example, 1975) in defending a pragmatist theory of vagueness that is supposed to retain a classical logic and semantics. I do not fully understand this approach, but my impression is that insofar as it acknowledges the existence of borderline cases, it collapses into the supervaluationist theory (see Keefe 2000, chap. 6, for instructive discussion). Unger (for example, 1979) also retains a classical logic and semantics for vague predicates, but at the cost of denying the existence of borderline cases: on his view, vague words are referentially vacuous. I will be discussing only views that acknowledge the existence (or at least the possibility) of borderline cases.

[5] See Keefe and Smith 1997, 1–57 for a guide to different versions of the standard analysis.

[6] Of course there may be other reasons, independent of the definition of borderline cases, to reject a classical logic and/or semantics for vague words. I don't think there are, but for all I will say here, there may be.

[7] For convenience, I will sometimes talk about colors instead of hues, but I mean always to refer to the single psychophysical dimension of hue, not the three-dimensional property of color. Nothing essential to my position turns on

this.

[8] An anonymous referee for this journal observes that there could be an ordering of window panes progressing from a red one to a transparent (hence colorless) one. The panes in the middle of the series might count as borderline cases for 'red'.

[9] Given my definition of 'Φ-ordering', I do not say that the ordering on dimension D, such as height, is a height-ordering (D-ordering). A height-ordering would be an ordering from an item that has a height to an item that does not. Rather, the ordering for 'tall' on the decisive dimension of height is an ordering *of heights*, in other words, an ordering of items all of which have a height. Similarly, our red-ordering is an ordering of hues, and our rich-ordering is an ordering of incomes. For ease of discussion, I will suppose that the ordering on D is linear, but I do not know whether all vague predicates require this. In the standard examples like 'red', 'tall', 'rich', 'bald', and 'heap', the ordering on D appears to be linear. For example, the items in a red-ordering that defines borderline cases for 'red' are linearly ordered with respect to hue as well as redness; the items in a rich-ordering that defines borderline cases for 'rich' are linearly ordered with respect to (say) dollar amount as well as richness, and so on.

[10] The argument from accessibility comes from an anonymous referee for this journal.

[11] Here especially I am indebted to Terry Horgan for helpful discussion.

[12] The kind of justification invoked here is not epistemic. Rather, I have in mind the sort of "conceptual" or "semantic" justification that is involved when we say that Jane's being a female sibling of John justifies the claim that she is his sister, or that the premises of an argument justify its conclusion.

[13] The symbol '*' is not used here as a functor. Rather, 'red*' is a variable that stands in for the various incompatibles of 'red', like 'orange', 'red-orange', 'green', 'pink', and so forth; 'rich*' stands in for 'middle class', 'upper middle class', 'poor', and so on.

[14] Thanks to John Collins for requesting this clarification.

[15] Since on the Incompatibilist View any borderline cases for 'not-rich' would be not-not-rich, (i.e., rich), 'not-rich' probably has no borderline cases. (How could a rich income be a borderline case for 'not-rich'?) It is an interesting question why 'rich' should have borderline cases while 'not-rich' does not. I cannot address it here, but the heterogeneous character of the extension of 'not-rich', even within a given not-rich-ordering, is surely significant; see my *Vagueness Without Paradox* (in progress; hereafter *VP*), chap. 6, for elaboration.

What about a predicate like 'red or orange', for example? Can it have borderline cases? Again, as far as the incompatibilist is concerned, 'red or orange' has borderline cases insofar as it has proximate incompatibles; but whether it does have proximate incompatibles is a question she leaves to competent speakers. For instance, is 'yellow' a proximate incompatible of 'red or orange' ('red-or-orange')? Can there be items that belong to a red-or-orange/yellow ordering but are neither red-or-orange nor yellow? I don't know the answer.

[16] Wright may endorse such a view in the following remarks :

> [W]herever a stable consensus can be elicited that something is on the border-

> line between two concepts, that is merely an indication that we could, if we wished, employ a concept intermediate between them and hence that they are not really complementary. (1995, 138)

The concepts Wright appears to have in mind are pairs like 'red' and 'orange', or 'rich' and 'middle class'; shortly thereafter he recommends that we conceive

> being borderline as a status consistent with both the polar verdicts: for an item to be a borderline case on the red-orange border is for it to have a status consistent both with being red and with being orange (so not red). … (139)

[17] As an incompatible of 'red', a predicate like 'red-orange' is to be distinguished from predicates like 'scarlet' and 'vermilion', which name subcategories or determinates, not incompatibles, of red. Presumably the proposed disjunction would not include the latter terms. I am grateful to Rob Koons and Nicholas Asher for discussion of this point.

A bit of trivia illustrates that terms like 'red-orange' are used in ordinary language. A color called 'red orange' was introduced into the Crayola crayon box in 1949—in addition to the colors red and orange—and to the best of my knowledge remains in the box of 120 colors available today. (Other examples are orange yellow, yellow orange, yellow green, blue green, and red violet.) The box also included a further distinct color called 'orange red' from 1949 until its retirement in 1990. Perhaps red[orange] borderline cases that are more red than orange should be called 'red orange', and those that are more orange than red, 'orange red'. But this of course is a question for competent speakers.

[18] See, for example, Burns 1995, 29. (Then see Keefe 2000, 208–10, for persuasive argument that this common conception of higher-order borderline cases cannot be right.) It is worth noting that if the possibility of such a hierarchy is what constitutes, or anyway provides for, the blurred boundaries of a vague predicate, then there are sharp boundaries in any sorites series. A sorites series is by definition finitely membered, so it cannot possibly contain such a hierarchy. Since I don't believe that higher-order borderline cases are necessary for blurred boundaries, this result does not trouble me; but I am puzzled at the lack of acknowledgment of it by proponents of the Standard Analysis$_s$.

[19] Indeed, advocates of the Standard Analysis$_s$ may be hard pressed to explain why their higher-order borderline cases are not just so many Φ, not-Φ, and "first-order" borderline items.

[20] Remember that rich[middle class] borderline incomes are (rich[middle class]) *not-rich.*

[21] Here one is reminded of Dummett:

> [T]here is no definite line between hills and mountains. But we could not eliminate this vagueness by introducing a new predicate, say 'eminence', to apply to those things which are neither definitely hills nor definitely mountains, since there would still remain things which were neither definitely hills nor definitely eminences, and so *ad infinitum.* (1981, 182)

[22] Sainsbury writes that "subjects asked to classify a range of test objects using just 'young' and 'old' make different assignments to these words from those they make to them when asked to classify using, in addition, 'middle-

aged'" (1997, 259). The following remarks by C. L. Hardin suggest that a similar dynamic operates among the hue predicates:

> [T]he boundary of red in the broadest sense extends to the immediate neighborhood of unique yellow, and the breadth of that spread we acknowledge by our use of the modifier 'reddish'. But, in a somewhat narrower sense, the boundary between red and yellow falls at the point at which the perceptual "pull" of yellow is equal to that of red. This point is, of course, orange. But once we introduce orange as a distinct hue category, its boundary with red is at issue, and the extension of 'red' must be contracted to make room for the oranges. The natural red-orange boundary would seem to fall at the 75 percent red, 25 percent yellow region which was well within the scope we took 'red' to have when we were concerned to compare red with yellow. (1988, 184)

[23] I elaborate in *VP*, chaps. 3 and 4.

[24] See Sainsbury 1997, 255. Virtually all theories of vagueness arrive ultimately at some tripartite classification or other (which may or may not result in sharp boundaries). Most theories must confront the fact that, on pain of incoherence, there can be no borderline cases between the items of which '*x* is Φ' is true and the items of which '*x* is Φ' is untrue, where by 'untrue' I mean *anything other than true*. Even Sainsbury, who aims to abolish boundaries altogether by conceiving of satisfaction of a vague predicate on the model of attraction to a magnetic pole, allows that "some objects cluster firmly to one pole, some to another, and some, though sensitive to the forces, join no cluster" (1997, 258).

[25] It is worth noting that the incompatibilist can agree with the metalinguistic thesis, endorsed by some proponents of the Standard Analysis$_S$ (for example, Wright (1995), Shapiro (2006)), that in at least some instances of judging something to be a borderline case for 'rich' (for example), one does not thereby commit oneself to supposing that a judgment of 'rich' or of 'middle class' is mistaken. Similarly, in at least some instances of judging something rich, one is not thereby committed to supposing that a judgment of 'borderline rich' or of 'middle class' is mistaken. Unlike Wright and Shapiro, however, I do not import the idea of permissible disagreement into the analysis of a borderline case; see again *VP*, chap. 6, for discussion.

[26] It will follow that if a predicate, say 'not-rich', can have no borderline cases, then to say that an item is definitely not-rich is, strictly speaking, pragmatically inappropriate. (Of course, one might actually have in mind that the item is definitely rich*, for some not readily available proximate incompatible 'rich*'; see the subsequent discussion on p. 23.)

[27] To deny the existence of higher-order borderline cases is not to deny the existence of higher-order vagueness altogether. Among other things, the language in which a theory of vagueness is expressed, for example, English, is surely vague. I discuss higher-order vagueness at some length in *VP*, chap. 6.

[28] I have avoided talk of our *knowing* that a borderline case is not-Φ and not-Φ*. This is not because I think that any facts about borderline cases are hidden from us, but rather because certain features of the semantics of vague predicates call into question whether our "access" to borderline cases is properly called 'knowledge'. I take up the matter in *VP*, chap. 4.

[29] See, just for example, Wright 1997, 228–34; Heck 1993; Keefe 2000, 26–36, 208–11; Graff 2003.

[30] Note the talk of uncertainty as a feeling.

[31] The supposed need to accommodate higher-order borderline cases is often used as a club by one side or another in disputes over the nature of borderline cases; see Williamson 1994, 156–61, and Keefe 2000, 112ff., for example.